NAG Toolbox for MATLAB

g02dn

1 Purpose

g02dn gives the estimate of an estimable function along with its standard error.

2 Syntax

```
[est, stat, sestat, t, ifail] = g02dn(irank, b, cov, p, f, tol, 'ip', ip)
```

3 Description

g02dn computes the estimates of an estimable function for a general linear regression model which is not of full rank. It is intended for use after a call to g02da or g02dd. An estimable function is a linear combination of the parameters such that it has a unique estimate. For a full rank model all linear combinations of parameters are estimable.

In the case of a model not of full rank the functions use a singular value decomposition (SVD) to find the parameter estimates, $\hat{\beta}$, and their variance-covariance matrix. Given the upper triangular matrix R obtained from the QR decomposition of the independent variables the SVD gives

$$R = Q_* \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} P^{\mathrm{T}},$$

where D is a k by k diagonal matrix with nonzero diagonal elements, k being the rank of R, and Q_* and P are p by p orthogonal matrices. This gives the solution

$$\hat{\beta} = P_1 D^{-1} Q_{*_1}^{\mathrm{T}} c_1,$$

P_1 being the first k columns of P, i.e., $P = (P_1 P_0)$, Q_{*_1} being the first k columns of Q_*, and c_1 being the first p elements of c.

Details of the SVD are made available in the form of the matrix P^*:

$$P^* = \begin{pmatrix} D^{-1} P_1^{\mathrm{T}} \\ P_0^{\mathrm{T}} \end{pmatrix},$$

as given by g02da and g02dd.

A linear function of the parameters, $F = f^{\mathrm{T}}\beta$, can be tested to see if it is estimable by computing $\zeta = P_0^{\mathrm{T}} f$. If ζ is zero, then the function is estimable; if not, the function is not estimable. In practice $|\zeta|$ is tested against some small quantity η.

Given that F is estimable it can be estimated by $f^{\mathrm{T}}\hat{\beta}$ and its standard error calculated from the variance-covariance matrix of $\hat{\beta}$, C_β, as

$$\mathrm{se}(F) = \sqrt{f^{\mathrm{T}} C_\beta f}.$$

Also a t-statistic,

$$t = \frac{f^{\mathrm{T}}\hat{\beta}}{\mathrm{se}(F)},$$

can be computed. The t-statistic will have a Student's t-distribution with degrees of freedom as given by the degrees of freedom for the residual sum of squares for the model.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Hammarling S 1985 The singular value decomposition in multivariate statistics *SIGNUM Newsl.* **20 (3)** 2–25

Searle S R 1971 *Linear Models* Wiley

5 Parameters

5.1 Compulsory Input Parameters

1: **irank – int32 scalar**

k, the rank of the independent variables.

Constraint: $1 \leq$ **irank** $\leq$ **ip**.

2: **b(ip) – double array**

The **ip** values of the estimates of the parameters of the model, $\hat{\beta}$.

3: **cov(ip × (ip + 1)/2) – double array**

The upper triangular part of the variance-covariance matrix of the **ip** parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in $\mathbf{b}(i)$ and the parameter estimate given in $\mathbf{b}(j)$, $j \geq i$, is stored in $\mathbf{cov}(j \times (j-1)/2 + i)$.

4: **p(ip × ip + 2 × ip) – double array**

As returned by g02da or g02dd.

5: **f(ip) – double array**

f, the linear function to be estimated.

6: **tol – double scalar**

η, the tolerance value used in the check for estimability.

tol ≤ 0.0

$\sqrt{\epsilon}$, where ϵ is the ***machine precision***, is used instead.

5.2 Optional Input Parameters

1: **ip – int32 scalar**

Default: The dimension of the arrays **b**, **f**. (An error is raised if these dimensions are not equal.)

p, the number of terms in the linear model.

Constraint: **ip** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

wk

5.4 Output Parameters

1: **est – logical scalar**

Indicates if the function was estimable.

est = **true**

The function is estimable.

est = **false**

The function is not estimable and **stat**, **sestat** and **t** are not set.

2: **stat – double scalar**

If **est** = **true**, **stat** contains the estimate of the function, $f^{\mathrm{T}}\hat{\beta}$.

3: **sestat – double scalar**

If **est** = **true**, **sestat** contains the standard error of the estimate of the function, $\mathrm{se}(F)$.

4: **t – double scalar**

If **est** = **true**, **t** contains the t-statistic for the test of the function being equal to zero.

5: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g02dn may return useful information for one or more of the following detected errors or warnings.

ifail = 1

On entry, **ip** < 1,
or **irank** < 1,
or **irank** $>$ **ip**.

ifail = 2

On entry, **irank** = **ip**. In this case **est** is returned as true and all statistics are calculated.

ifail = 3

Standard error of statistic $= 0.0$; this may be due to rounding errors if the standard error is very small or due to mis-specified inputs **cov** and **f**.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The value of estimable functions is independent of the solution chosen from the many possible solutions. While g02dn may be used to estimate functions of the parameters of the model as computed by g02dk, β_c, these must be expressed in terms of the original parameters, β. The relation between the two sets of parameters may not be straightforward.

9 Example

```
irank = int32(4);
b = [30.55666666666667;
     5.446666666666673;
     6.743333333333336;
     11.04666666666667;
```

```
     7.32];
cov = [0.1481786666666657;
     0.03704466666666638;
     0.7038486666666622;
     0.03704466666666652;
     -0.2222679999999989;
     0.7038486666666626;
     0.03704466666666648;
     -0.2222679999999987;
     -0.2222679999999985;
     0.7038486666666621;
     0.03704466666666641;
     -0.2222679999999984;
     -0.2222679999999987;
     -0.2222679999999985;
     0.7038486666666622];
p = [1.135198279858991;
     1.13078348304906;
     1.234006904805161;
     1.228003039942231;
     1.263332232672042;
     3.872983346207417;
     1.732050807568878;
     1.732050807568878;
     1.732050807568877;
     1.64780261914309e-16;
     0.2309401076758503;
     8.35089227579159e-18;
     -1.238223776129407e-17;
     7.900875770685859e-17;
     -0.447213595499958;
     0.0577350269189626;
     0.2871305767251709;
     0.2022129403845741;
     -0.3559016137228031;
     0.447213595499958;
     0.05773502691896256;
     0.03726461792122844;
     0.271286303307133;
     0.4183480487454616;
     0.447213595499958;
     0.05773502691896257;
     0.1515575558927682;
     -0.4677758485215264;
     0.0906424999201606;
     0.447213595499958;
     0.05773502691896258;
     -0.4759527505391677;
     -0.005723395170180857;
     -0.153088934942819;
     0.4472135954999578];
f = [1;
     1;
     0;
     0;
     0];
tol = 1e-05;
[est, stat, sestat, t, ifail] = g02dn(irank, b, cov, p, f, tol)
```

```
est =
     1
stat =
   36.0033
sestat =
    0.9623
t =
   37.4119
ifail =
           0
```